Shell model study of the pairing correlations

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A systematic study of the pairing correlations as a function of temperature and angular momentum has been performed in the sd-shell region using the spherical shell model approach. The pairing correlations have been derived for even-even, even-odd and odd-odd systems near N=Z and also for the asymmetric case of N=Z+4. The results indicate that the pairing content and the behavior of pair correlations is similar in even-even and odd-mass nuclei. For odd-odd N=Z system, angular momentum I=0 state is an isospin, t=1 neutron-proton paired configuration. Further, these t=1 correlations are shown to be dramatically reduced for the asymmetric case of N=Z+4. The shell model results obtained are qualitatively explained within a simplified degenerate model.

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I. INTRODUCTION

It is well established that the pairing field is an important component of the nuclear mean-field potential. The interplay between the deformation driving forces and the pairing field determines most of the properties of a nuclear system. The relevance of the pairing field for the nuclear many-body system was proposed in the pioneering work of Bohr, Mottelson and Pines in 1958 [1]. There are two issues related to the pairing potential which still need to be elucidated. The first issue concerns with the detailed form of the pairing potential and the second to the approximation employed for the solution of pairing force. Although, it is quite evident from the analysis of the properties of nuclei, for instance the suppression of the moments of inertia of rotating nuclei and the observed energy gaps, that pairing field is essential for describing atomic nuclei [2, 3]. But most of the properties of nuclei are rather insensitive to the detailed form of the potential in the pairing or the particle-particle channel. In comparison, most of the nuclear properties such as compressibility, surface energy, effective masses, saturation and other specific properties critically depend on the exact form of the nuclear potential in the Hartree Fock or particle-hole channel. In addition, the potential in the particle-hole channel is constrained or adjusted to the properties of the closed shell nuclei and the pairing field is introduced in an ad hoc manner in most of the density functional theories (DFT). In most of these approaches, for instance Skyrme and relativistic mean field models, the potential is chosen to be of zero range and therefore, in principle, don't contain pairing correlations. In the finite range Gogny density functional approach, the range of the effective pairing force is adjusted to the properties of G-matrix calculated from the bare nucleon-nucleon interaction.

The second issue concerns with the method employed to solve the pairing interaction. The pairing field has been primarily studied in the Bardeen-Cooper-Schrieffer (BCS) or Hartree-Fock-Bogoliubov (HFB) approximation. In this approach, pairing field depicts a sudden transitional behavior as a function of rotational frequency and temperature [4, 5]. The pairing correlations are finite up to a certain rotational frequency or temperature and then these correlations suddenly vanish above this transitional point. The empirical analysis of the experimental data, however, does not depict this sudden phase transition and shows a smooth transition from one phase to the other. The reason for this discrepancy is known to arise from the neglect of the fluctuations in the mean-field models [6, 7, 8, 9, 10].

It is known that relative size of the fluctuations becomes small for a system with a very large number of particles, for example a bulk superconductor, and sudden transitional behavior is a hall mark of these macroscopic systems. However, for systems with finite number of particles, for instance atomic nuclei and metallic clusters, the fluctuations are important and need to be incorporated for an accurate description of these systems [11, 12]. A powerful method to incorporate the fluctuations is the projection theory. In particular, the particle number projection methods are now readily available to incorporate the fluctuations quite accurately. However, these projection methods, before variation, are available only at zero temperature [9, 13, 14, 15, 16] in the complete HFB framework. It needs to be added that the particle number projection analysis has been performed at finite temperature with BCS ansatz [17, 18].

The projection at finite temperature is more important as it is known that the mean-field or BCS wavefunction for an even or odd particle system has admixtures from both even and odd neighboring particle numbers [12, 19]. At zero temperature, even (odd) system has admixtures from only even (odd) particle numbers. It has been also demonstrated recently, in an exactly solvable model, that the pairing correlations re-appear at finite temperature after they are quenched at zero temperature and high rotational frequency [8, 20]. This surprising result, which completely contradicts the mean-field predictions, needs to be studied in more realistic models.

The spherical shell model (SSM) is another tool to study pairing correlations. The advantage of the SSM

approach is that, first of all, most of the interactions employed in SSM are adjusted to open shell nuclei and, therefore, contain a proper pairing force. Secondly, the exact diagonalization of the Hamiltonian matrix is performed and results obtained contain all the possible correlations including the pairing. There are many puzzling questions regarding the pairing correlations which can be addressed within the framework of SSM. For instance, difference between the pairing correlations in odd- and even-mass nuclei, importance of the neutron-proton pairing correlations near N=Z and the phase transition mentioned above. These issues originate when comparing the mean-field results with the experimental data or exact solutions of toy models. It needs to mentioned that in SSM approach only low-lying part of the excitation spectrum is fitted and, therefore, the results obtained using SSM are inaccurate for high-excitation energy or temperature.

The purpose of the present work is to study the pairing correlations in a realistic space using the spherical shell model approach. It needs to be emphasized here that a new shell model program has been developed by two of the present authors [21]. This new program completely works in the j-representation [22] and is quite similar in structure to that of NATHAN code developed by the Madrid-Strasbourg group [23]. The shell model approach provides the most accurate description of nuclear properties and incorporates all the possible correlations. However, the SSM analysis is restricted to lighter nuclei and its application to heavier nuclei appears impossible in the near future. The most recent progress in the SSM approach is the study of the fp-shell nuclei [23].

In the present work, the shell model analysis has been performed in the sd-shell region. The reason for choosing sd-shell is that, first of all, the interaction is well established and it is known that "USD" interaction [24] provides an accurate description of most of the sd-shell nuclei. Secondly, it is required, in the present analysis, to calculate a few thousand states for each angular momentum to evaluate the statistical partition function. These calculations would become impossible in the fp-shell region. The calculations have been performed in the middle of the sd-shell so that a large number of eigen-solutions are available for the statistical analysis. The nuclei for which the shell model calculations have been performed are : ^{28}Si, ^{27}Si and ^{26}Al. These three neighboring nuclei have been investigated so that a comparison can be made among even-even, even-odd and odd-odd systems. The pairing correlations have also been deduced for the asymmetric system ^{24}Ne in order to study the dependence of the neutron-proton pairing on the particle number.

It is pertinent to mention here that while the present work was in progress, a similar study of pairing correlations in the sd-shell has been recently published [25]. Although the model and the region of study in the present work is same as that of ref. [25], the issues discussed are different and as a matter of fact the present work complements the work of ref. [25]. In the course of the discussion, we shall comment on the results of ref. [25] wherever necessary. The present manuscript is organized as follows : In the next section, the shell model based expressions are presented for the evaluation of the pairing correlations. The results of the calculations are presented and analyzed in section III and finally the summary and conclusions are included in section IV.

II. SHELL MODEL FORMULATION

The spherical shell model Hamiltonian, generally, contains single-particle and two-body parts and in the second quantized notation is written as

$$\hat{H} \;=\; \hat{h}_{sp} \,+\, \hat{V}_2, \tag{1}$$

where,

$$\hat{h}_{sp} \;=\; \sum_{rs} \epsilon_{rs}\, c_r^\dagger\, c_s, \tag{2}$$

and

$$\begin{aligned}\hat{V}_2 \;&=\; \frac{1}{4}\sum_{rstu} < rs|v_a|tu > c_r^\dagger c_s^\dagger c_u c_t \\ &=\; \sum_{rstu\Gamma} \frac{\sqrt{(2\Gamma+1)}}{\sqrt{(1+\delta_{rs})(1+\delta_{tu})}} < rs|v_a|tu >_\Gamma \\ &\qquad \left(A_\Gamma^\dagger(rs) \times \tilde{A}_\Gamma(tu)\right)_0,\end{aligned} \tag{3}$$

where ϵ_{rs} are the single-particle energies of the spherical shell model states, which are diagonal except in the radial quantum numbers and $< rs|v_a|tu >_\Gamma$ are the two-body interaction matrix elements and in the present work are chosen to be those of "USD". The two-particle coupled operator in Eq. (3) is given by $A_\Gamma^\dagger(rs) = (c_r^\dagger c_s^\dagger)_\Gamma$ and $\tilde{A}_{\Gamma M_\Gamma} = (-1)^{\Gamma - M_\Gamma} A_{\Gamma - M_\Gamma}$. The labels $r, s, ...$ in the above equations denote the quantum numbers of spin and isospin. "Γ" quantum number labels both angular momentum and isospin of the coupled state. The above notation is same as that used in ref. [22].

In the present work, the pairing correlations have been calculated using the canonical ensemble approach since the exact solutions have well defined particle number. The average value of a physical quantity "F" in canonical ensemble is given by [8, 11]

$$<< F >>= \sum_i F_i e^{-E_i/kT}/Z, \tag{4}$$

where,

$$Z \;=\; \sum_i e^{-E_i/kT} \tag{5}$$

$$\begin{aligned}\hat{H}|i> \;&=\; E_i|i> \\ F_i \;&=\; < i|\hat{F}|i>\end{aligned} \tag{6}$$

In the partition function, Eq. (5), k is the Boltzman constant and T is the temperature. In the rest of the manuscript, we shall use "Temp" rather than kT, which has dimensions of energy (MeV) and the symbol T shall be used for the isospin of the coupled two-particle state.

In the present work, we study the isovector monopole pair correlations for the "$\Gamma_0 = \{I = 0, T = 1, |T_z| = 0, 1\}$". The canonical average for this is calculated as

$$E_{\Gamma_0}(\text{pair}) = \sum_{rstu} \frac{\sqrt{(2\Gamma+1)}}{\sqrt{(1+\delta_{rs})(1+\delta_{tu})}} < rs|v_a|tu >_{\Gamma_0} << (A^\dagger_{\Gamma_0} \times \tilde{A}_{\Gamma_0})_0 >>, \tag{7}$$

from the energies and eigen-states obtained by diagonalization, which shall be referred to as the "correlated" pairing energy $E_{pair}(corr)$. We have also calculated the "uncorrelated" pairing energy by removing the monopole field Γ_0 in the interaction and then redoing the shell model diagonalization [8]. Using the resulting energies and wavefunctions, Eq. (7) is evaluated to give the "uncorrelated" pairing energy. It is noted that the definition of present pairing energy is slightly different from that used in refs. [25, 26]. In these studies, it is calculated without the matrix elements and the constant factors in front of the expectation values in Eq. (7).

III. RESULTS AND DISCUSSIONS

The shell model calculations have been performed in the middle of the sd-shell for ^{28}Si (with six valence protons and six valence neutrons), ^{27}Si (with six valence protons and five valence neutrons) and ^{26}Al (with five valence protons and five valence neutrons). The calculations have also been performed for ^{24}Ne (with two protons and six neutrons) in order investigate the behavior of the pairing correlations with asymmetry in proton and neutron particle numbers. The temperature and the angular momentum dependence of the pair correlations have been studied in detail for these systems and are discussed in subsections B and C. In subsection D, the temperature dependence of the average isospin is discussed. Before presenting these results, in subsection A a simple model is briefly described to analyze the essential results obtained with the full shell model calculations.

A. Isospin geometry and a simple model

Before starting the discussion, we first briefly recall some consequences of isospin being a good quantum number. Writing explicitly the isospin and omitting the other quantum numbers, i.e. $\Gamma = T, T_z$, the pair energy is measured by

$$A^\dagger_{1,M} \times \tilde{A}_{1,-M} = \sum_{t=0,1,2} < 1M1-M|t0 > B_{t0}, \tag{8}$$

where the proton-neutron paring corresponds to $M = 0$ and the neutron-neutron pairing to $M = 1$. Since $A^\dagger_{1,M}$ and $\tilde{A}_{1,-M}$ are tensor operators in isospace their product can be rewritten as a sum of tensor operators B_{t0} and

$$\begin{aligned} < TT_z|A^\dagger_{1,M} &\times \tilde{A}_{1,-M}|TT_z > \\ &= \sum_{t=0,1,2} < 1M1-M|t0 > \\ &\times < TT_z10|TT_Z >< T||B_t||T > . \end{aligned} \tag{9}$$

Only the $t = 0$ term contributes in $T = 0$ states, which means that $< T = 0|A^\dagger_{1,M} \times \tilde{A}_{1,-M}|T = 0 >$ does not depend on M, i. e. E_{pair} is the same for all three types of pairing. For $T > 0$ states, the terms $t = 1, 2$ contribute as well, which means E_{pair} may be different for each pairing channel.

For a quantitative statement, one needs to know the reduced matrix elements $< T||B_t||T >$, which depend on the detailed structure of the shell model states. For the lowest states, pairing correlations are strong. In such a case one expects that the model of isovector monopole pairing in a degenerate shell should allow us some rough estimate of the relative paring strengths. The model is discussed in [27, 28], where the original work is cited and explicit expressions are given in terms of number of nucleon pairs $\mathcal{N}$ in the shell, the isospin T, T_Z, and the number of pairs $\Omega(= 12)$ that the shell can accommodate, where the unit is the coupling constant G. It was found there that the model accounts well for relative strengths of the pair energies but cannot reproduce the scale of the shell model calculations, which was attributed to the splitting of the levels due to the deformation and the spin-orbit coupling. Following this observation, we scale the pair energies of the degenerate model by a common factor.

B. Temperature dependence of the pair correlations

The results of the neutron-neutron and neutron-proton monopole pair correlation energies for $^{28}_{14}$Si$_{14}$ are shown as a function of temperature (Temp) in Figs. 1 and 2 for even- and odd-spin values. For the symmetric system and isospin invariant two-body interaction, the proton-proton pairing energy is identical to that of neutron-neutron. The reason is that the canonical ensemble contains mainly isospin, $T = 0$ states, which lie lower than the $T > 0$ states in the N=Z=even nuclei [29].

Even- and odd-spin values are plotted separately in two figures since they have different intrinsic structures. In the quasi-particle language, the low-lying even-spin members in an even-even nucleus have 0-quasiparticle intrinsic structure and the odd-spin members have 2-quasiparticle structure. Obviously, the fraction of $T > 0$ states is larger for odd spin.

In order to investigate, in detail, the variation of the pairing correlations with spin (I), the correlations are

plotted separately for each possible value of spin and are shown in two panels. In the top panel the pairing correlations are calculated with full two-body interaction [denoted by $E_{pair}(corr)$]. The lower panel depicts pairing correlations [denoted by $E_{pair}(uncorr)$] in which the monopole terms in the two-body interaction were excluded.

For $I = 0$ in Fig. 1, the $E_{pair}(corr)$ are quite similar for neutron-proton and identical particle channels and are also quite large at low temperatures. For Temp=0, they are 4.6 MeV. The ground-state in even-even systems is a paired configuration with maximum correlations. The degenerate model gives 3/2 for the correlation energy, which fixes the scale factor to 3.06 MeV. In the following we shall use this factor to scale other calculations in the framework of the degenerate model.

The pair correlations are almost unchanged till Temp$\simeq$2 MeV and then are observed to be reduced with increasing temperature. However, in comparison to the mean-field models which predict a sudden transition from the paired to the unpaired state, the exact analysis depicts a smooth drop in the pair correlations. The phase transition obtained at Temp$\simeq$2 MeV in Fig. 1 is higher as compared to the transition point obtained in HFB and also in shell model Monte-Carlo study, which predict at Temp$\simeq$0.5 to 0.7 MeV [30]. It is to be noted that in the present work, the pairing correlations have been obtained using the expression, Eq. (7) and, as already pointed out at the end of section II, is slightly different from the expression used in the HFB and the shell model Monte-Carlo studies. In order to confirm that the reason for the discrepancy in the phase transition point is due to the different pairing expressions used, we have performed shell model calculations for a simpler case of ^{24}Mg with the pairing expression as used in HFB and Monte-Carlo studies and the phase transition was found to be around 0.9 MeV. It is also noted that the magnitude of the pairing energy obtained in the present study is also quite large as compared to the HFB and the shell model Monte-Carlo results. Further, it is seen in Fig. 1 that at large temperatures the neutron-proton pairing energy deviates from proton and neutron pairing energies, which was also observed in ref. [26]. The difference between the proton-neutron and neutron-neutron (=proton-proton) pair energies indicates some admixture of $T = 1$ states to the ensemble.

For even-spin values of I=2, 4 and 6, it is observed that the pair correlations drop in a step wise manner and the pairing gaps for these spin values at Temp=0 are approximately 3.9, 3.2 and 2.6. The temperature dependence of the pair energies for $I = 2$ and 4 show a similar behavior as that of $I = 0$. For I=6, the pair correlations are almost constant with temperature. The shell model calculations for $I = 8$ and 10 (not shown in the figure) depict a slight increase in the pair correlations at low temperatures and for higher temperatures the correlations drop as for the earlier cases.

The uncorrelated pairing energy are shown in the lower panels of Fig. 1. As already mentioned, they have been obtained by a second shell model diagonalization on setting the monopole matrix elements equal to zero. The problem in the shell model study of the pair correlations is that the calculated correlated pairing energy may contain the contributions from other multipoles due to recoupling, which in the mean-field language are referred to as the particle-hole contribution. In the mean-field framework, particle-particle and the particle-hole channels are completely decoupled and the pairing energy can be directly evaluated from the pairing potential.

It is noticed from Fig. 1 that the uncorrelated pairing energy is substantially smaller as compared to the correlated one. However, it shows a similar transitional behavior with temperature as the correlated pairing energy. In order to explore the reason for this unexpected transitional behavior in the uncorrelated energy, we have also removed I=1 and 2 apart from I=0 matrix elements in the shell model diagonalization and the pairing energy again calculated using the expression of Eq. (7). The results indicate that the phase transition is now almost washed out in the uncorrelated pairing energy. We are presently performing a detailed investigation of this phenomenon and the results of this study will be presented elsewhere.

The neutron-proton uncorrelated pairing energy appears to be lower than that of the corresponding neutron and proton energies. This can be understood as follows. The isovector monopole pair correlations shift the $T = 0$ states to lower energy as compared to the $T > 0$ states. If these correlations are switched off, then this preference is lifted. The increased fraction of $T > 0$ states in the ensembles creates the difference between the neutron-neutron and proton-neutron pairing. However, using this simple perspective, it is not possible to understand why the like-particle pairing is stronger.

The odd-spin values are depicted in Fig. 2. The pair energies are smaller than the even-spin values at very low temperature. This is easily understood by noting that the odd-spin band have two quasi-particle structure with reduced pairing. For $I = 1$, the yrast state has $T = 0$. The pronounced increase of the neutron-proton pair energy as compared to the like-particle ones indicates that low-lying $T > 0$ states become a substantial fraction of the ensemble.

For the larger angular momenta, $I = 7$ it is observed that the pair correlations first *increase* with temperature. It is well known that in the mean-field theory, the pair correlations are reduced with both increasing temperature and rotational frequency. The mean-field analysis always depicts a phase transition from the paired to the unpaired phase with increasing temperature and rotational frequency. This is experimentally observed in macroscopic superconductors, where the particle number is quite large. However, for mesoscopic systems the mean-field solution is inappropriate. The exact solutions of the pairing problem do not have a sharp phase transitions. Further, they may have peculiarities, as e.g.,

recently demonstrated in a simple few-level model that the pairing correlations re-appear at finite temperature after they are quenched at zero temperature and high rotational frequency [8], where this peculiar effect is explained. The shell model results for I =9 and 11 (not shown in the figure) also depict a minor increase. Therefore, increase in the pairing correlations with increasing temperature obtained in the earlier work is an artifact of the simple model employed [8] as the present results in the sd-shell don't depict such a dramatic rise. However, it needs to be added that in the sd-shell, it is not possible to have a large angular momentum where the re-appearance of the pairing correlations is predicted in the earlier simple model study.

The results of the pair energies for the odd-system ${}^{27}_{14}\mathrm{Si}_{13}$ are presented in Fig. 3. The results presented in this figure also correspond to ${}^{27}_{13}\mathrm{Al}_{14}$ with proton and neutron curves interchanged. At zero temperature the expected picture evolves. The proton-proton energy of 4.4 MeV is similar to the value of 4.6 MeV in the even-even neighbor ${}^{28}_{14}\mathrm{Si}_{14}$, as both have same proton number. The neutron-neutron pair energy of 3.0 MeV is about 30% lower due to the blocking of one level in the odd-neutron system. This is in accordance with the mean-field predictions that the pair gaps in an odd-system is reduced by about 15% in comparison to the neighboring even-even system, which amounts to a reduction of about 30% in the correlation energy. The np-pair energy of 3.7 MeV is less reduced than the neutron-neutron one. This is understood, because the odd neutron blocks the level only partially for proton-neutron pairs. [If the odd neutron is in the state $m>0$, a pair with the neutron in $m<0$ and the proton in $m>0$ may by accommodated.] The pair energies are different, which is expected for $T \leq 1/2$ states from Eq. (9). The degenerate model allows us to estimate the consequences of blocking. For $T=1/2$, $\mathcal{N}=4$ it gives $E_{pp}(corr)=3/2$, $E_{np}(corr)=15/12$, $E_{nn}(corr)=1$. The respective scaled values of 4.6, 3.8 and 3.1 MeV compare well with the shell model values of 4.4, 3.7 and 3.0 MeV.

In early nineties, it was demonstrated through a series of systematic studies [31] of experimental data that the difference between the moments of inertia of neighboring even-even and odd-nuclei is merely $\leq 2\%$. This posed a serious challenge to the traditional nuclear structure models based on mean-field theory which give a difference of about 15%. It was shown later using number projected mean-field models that this difference in the moments of inertia could be reduced. The moment of inertia depends on deformation and the pairing correlations and considering that ^{28}Si and ^{27}Si have similar deformation values, it is expected that the moments of inertia of two nuclei would be similar since the pair gaps of the two systems are similar.The calculated moments of inertia for low-lying states for ^{28}Si and ^{27}Si are 2.62205 $\hbar^2/(MeV)$ and 2.66220 $\hbar^2/(MeV)$, respectively.

The results for the odd-odd ${}^{26}_{13}\mathrm{Al}_{13}$ system are presented in Figs. 4. For low-spin, the pair energies are rather different from those of the even-even and odd-systems. For $I=0$, neutron-proton pairing energy is quite large as compared to like-particle pairing energies. The ground states in odd-odd self-conjugate nuclei show a preference for $T=1$ [29] with a $T=0$ state close by. In our case the ground state has $T=1$. For $T=1,\ T_z=0$, the simple model of isovector pairing in a degenerate shell gives a ratio $E_{pn}(corr)=43/15$, $E_{nn}(corr)=E_{pp}(corr)=11/15$. The respective scaled values of 7.8 and 2.2 MeV are to be compared with the shell model results of 7.9 and 2.7 MeV. The proton-neutron pair energy remains larger than the neutron-neutron pair energy for finite temperatures, which indicates that the isovector proton-neutron pair correlations lower the $T=1,\ T_z=0$ states relative to the $T=0,\ T_z=0$ states. The lowest $I=2$ state has $T=0$ and the same pairing energies for like and unlike particles. For higher temperature, neutron-proton pairing is larger as compared to identical particle pairing, which indicates a large fraction of $T=1$ states in the ensemble. With increasing spin the results become similar to the even-even system, which can be understood as a consequence of the quenching of the pair correlations in both types of nuclei. This is corroborated by the observation for the odd-A case that the different pair energies become similar (though not equal) at large spin. For odd-spin values, the pair energies are nearly equal for all spin values. Only at higher temperatures there is a slight asymmetry. This indicates that the odd-spin states have preferentially $T=0$.

The pair correlations for the asymmetric system ${}^{24}_{10}\mathrm{Ne}_{14}$ are presented in Fig. 5. As expected, the neutron pair energy is larger than the proton one, because there are more neutrons in the open shell. The neutron-proton pair energy is quite small. In contrast to the symmetric even-even system, where it is as large as the like particle one. This is in accordance with the HFB theory with neutron-proton pairing [32], which finds quite strong neutron-proton pairing for the N=Z system, but vanishing for the asymmetric case of N=Z+4. The present shell model substantiate these HFB results, although the neutron-proton has not vanished in the shell model study but has clearly become quite weak. The reason is that the extra neutron pair blocks a level for the proton-neutron pairs to scatter into. For zero temperature, the difference between the symmetric and asymmetric systems is qualitatively reproduced by the simple model of a degenerate shell. The ground state has $\mathcal{N}=4,\ T=2$, which gives $E_{pp}(corr)=13/14$, $E_{nn}(corr)=27/14$, $E_{np}(corr)=13/42$. The respective scaled values are 2.8, 5.9 and 1.0 MeV to be compared with the shell model results of 3.4, 5.5 and 1.2 MeV.

The model of a degenerate shell obeys the dynamical symmetry of the group SO(5) with the isospin subgroup SU(2). As will be discussed in a forthcoming paper, the relative strength of the three types of isovector pairing mainly reflects the geometry of the isospin induced by SU(2), which remains valid for the case of a

non-degenerate shell. The full SO(5) symmetry is exploited when the effects of blocking in states with non-zero seniority are estimated as for the case of odd-A in this paper.

C. Angular momentum dependence of the pair correlations

The angular momentum dependence of the pair correlations for the four systems studied are depicted in Fig. 6. The results for ^{28}Si indicate that for zero temperature, the pair correlations drop monotonically with increasing angular momentum, except for $I = 1$ which shows a larger drop. In the case of ^{27}Si, the pair correlations show a staggering effect, where the phase of the staggering is same for the three pairing modes.

For ^{26}Al, neutron-proton correlations are maximal for $I = 0$. This is evident from Fig. 6 with a drop of about 5 MeV from $I = 0$ to 1 in the correlated pairing energy. For the higher spin values, it is noted that pair correlations drop very little. The results for ^{24}Ne depict a larger staggering effect as compared to ^{28}Si for identical particle pairing. The neutron-proton pairing shows an irregular behavior with spin for this asymmetric system. As expected, for higher temperature of 3 MeV, the pair correlations depict a smoother behavior with spin.

D. Isospin analysis

In most of the analysis presented in the above subsections, the isospin content of the states played a crucial role to understand the behavior of the pairing energies. It is, therefore, imperative to ascertain the temperature dependence of the isospin. It needs to be mentioned that the new shell model program developed [21] uses the neutron-proton product basis and, therefore, isospin, although conserved, needs to be evaluated for each eigenstate. The expectation value of $\hat{T}^2$ has been calculated using the shell model wavefunctions as discussed in ref. [33]. The average value of T^2 as a function of temperature has been obtained by using the canonical partition function in the same manner as the pairing correlations have been deduced.

The average value of T^2 is shown in Fig. 7 for the lowest angular momentum ensemble. In the top panel of Fig. 7, average value of T^2 is plotted for ^{28}Si and it is quite evident from this figure that the isospin at low temperature is T=0 up to Temp=2 MeV. Above this temperature, the isospin increases steadily and correlates well with the drop in the pair energies observed in Fig. 1. For the odd-odd ^{26}Al system, the isospin at low temperature is equal to one and giving rise to large difference in the pairing correlations between neutron-proton and identical particle channels in Fig. 4. However, with increasing temperature, it is noted that the average isospin drops as T=0 states enter into the ensemble with the consequence that neutron-proton pairing energy comes closer to the identical particle pairing at higher temperatures in Fig. 4.

For the asymmetric case of ^{24}Ne, the average isospin in Fig. 7 is equal to two and appears to be almost constant with increasing temperature. This constancy of the average isospin gives rise to the constant behavior of the neutron-proton pairing energy in Fig. 5. The temperature behavior of the isospin for the odd-mass system ^{27}Si is similar to the even-even system ^{28}Si and is the reason that the behavior of the pairing correlations for the two systems are quite similar.

IV. SUMMARY AND CONCLUSIONS

In the present work, the shell model study of the pairing correlations has been undertaken. The calculations have been performed in the sd-shell for ^{28}Si, ^{27}Si, ^{26}Al, and ^{24}Ne. For the case of the even-even system ^{28}Si, the pair energy of even-spin states as a function of temperature depicts a smooth but pronounced decrease around Temp=2 MeV, which can be interpreted as the strongly washed out relic of the phase transition. In the case of the odd-spin states up to $I = 7$, the pairing correlations decrease only very slowly with increasing temperature, starting from a reduced value, which is caused by the blocking of two levels in the states carrying two quasi particle character.

It is also clearly evident from the present study that pairing correlations are non-zero even at large temperatures and angular momenta. This is in contradiction to the mean-field predictions that pairing correlations die out at higher temperatures and angular momenta. For the very small systems studied, most of the pair correlations is generated by the fluctuations of the pair field, which are more prominent than the mean-field itself.

The proton-proton, neutron-neutron, and proton-neutron isovector pair correlation energies are not scalar under rotation in isospace, which means that they are only equal in $T = 0$ states but generally different in $T > 0$ states. For the ground states of even-A nuclei, the relative strengths of the different pairing energies is qualitatively reproduced by the simple model of a degenerate shell. The even-even $N = Z$ nucleus has a T =0 ground state and thus equal proton-proton, neutron-neutron, and proton-neutron strength. The proton-neutron strength deviates from the identical ones with increasing temperature because $T > 0$ states enter the ensemble. The odd-odd $N = Z$ nucleus $^{26}_{13}\mathrm{Al}_{13}$ has a $T = 1,\ T_z = 0$ ground state for which the proton-neutron strength is about three times larger than the like particle strength. With increasing temperature, $T = 0$ states enter the ensemble, reducing the the proton-neutron contribution. The asymmetric nucleus $^{24}_{10}\mathrm{Ne}_{14}$ has a $T = 2, T_z = 2$ ground state with the proton-neutron pairing dramatically reduced as compared to the symmetric systems ^{28}Si and ^{26}Al. In the odd-neutron

nucleus $^{27}_{14}Si_{13}$, the proton-proton strength is about the same as in $^{28}_{14}Si_{14}$, the neutron-neutron strength is about 30% lower and the proton-neutron strength in between, which reflects the blocking of one level by the odd neutron.

Finally, we would like to mention that the results of the present work are questionable at higher temperatures as the configuration space of sd-shell employed in the present work is suited only for low-excitation energies. For higher temperatures, it is expected that fp-shell will be populated and for accurate evaluation of pairing correlations it is essential to include fp-shell configuration space in the shell model analysis. However, it is impossible to perform shell model calculations with a complete sdfp-configuration space. What is feasible is to calculate the partition function of the fp-shell in a spherical degenerate limit and then calculate the total partition function with the method illustrated in ref.[34]. We are presently working to evaluate the pairing correlations using this approach and the results of this analysis shall be presented in the near future.

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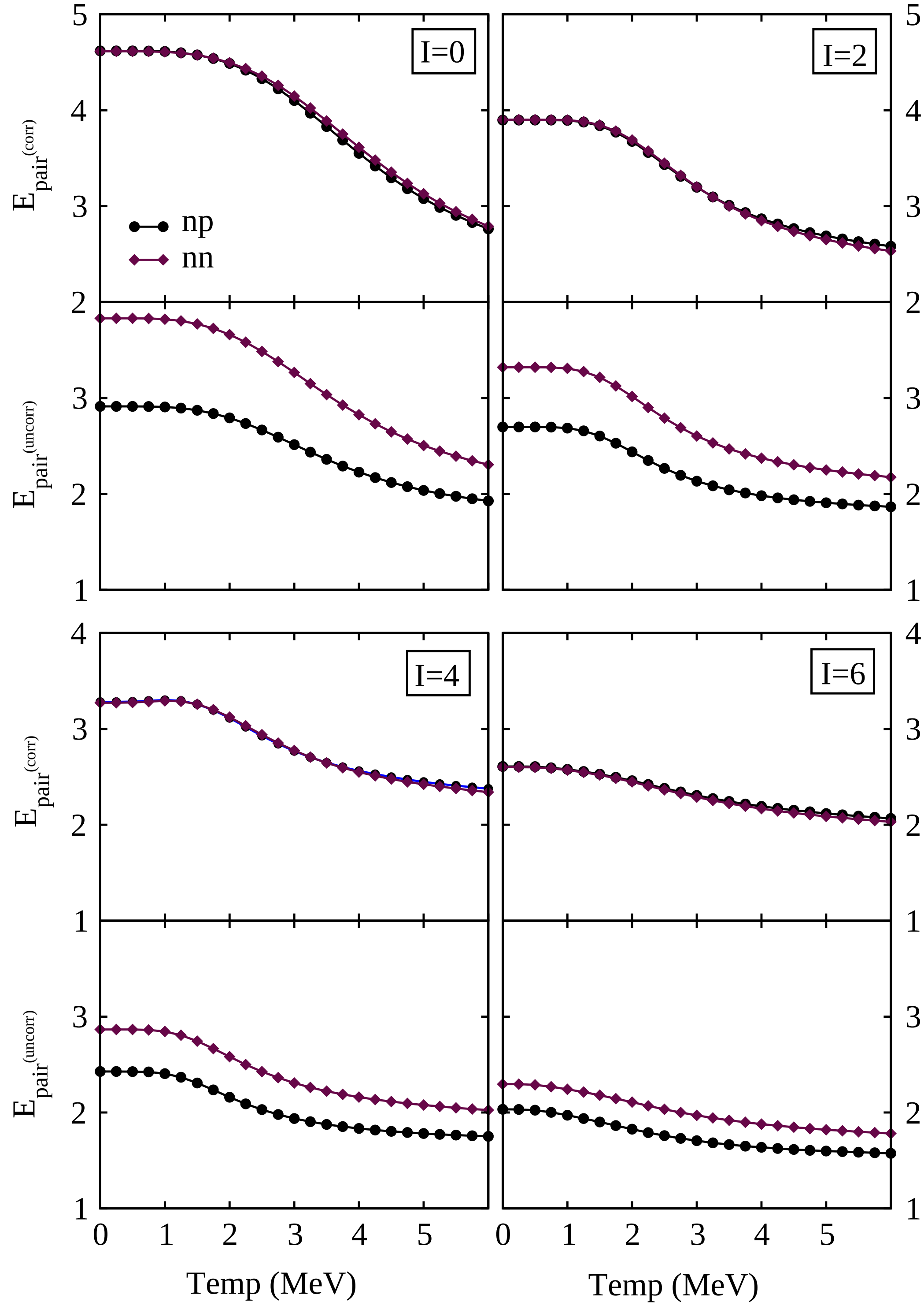

FIG. 1: (color online) Temperature dependence of pair correlations for ^{28}Si. The results are shown for even-spin values of I = 0, 2, 4, 6. For each spin state, there are two panels. The upper panel depicts the pairing correlations calculated with the full two-body interaction. The lower panel shows the pairing energy without monopole matrix elements, referred to as the uncorrelated contribution. For I=0, the temperatures of 1,2, 3, 4 and 5 MeV correspond to excitation energies of 0.04, 0.90, 4.0, 9.57 and 15.38 MeV, respectively.

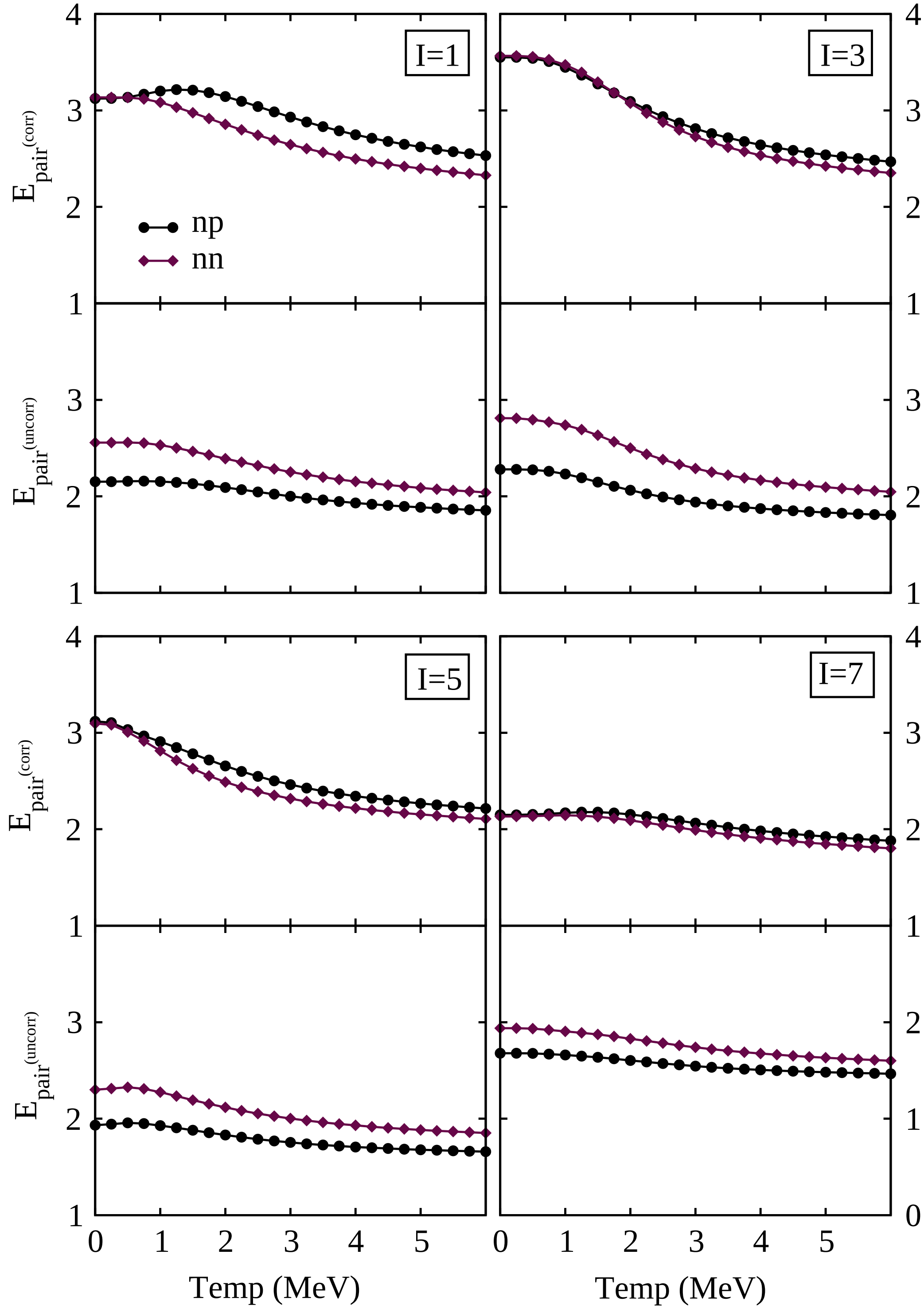

FIG. 2: (color online) Temperature dependence of pairing energy for ^{28}Si. Pair energy is shown for odd-spin values with I = 1, 3, 5 and 7.

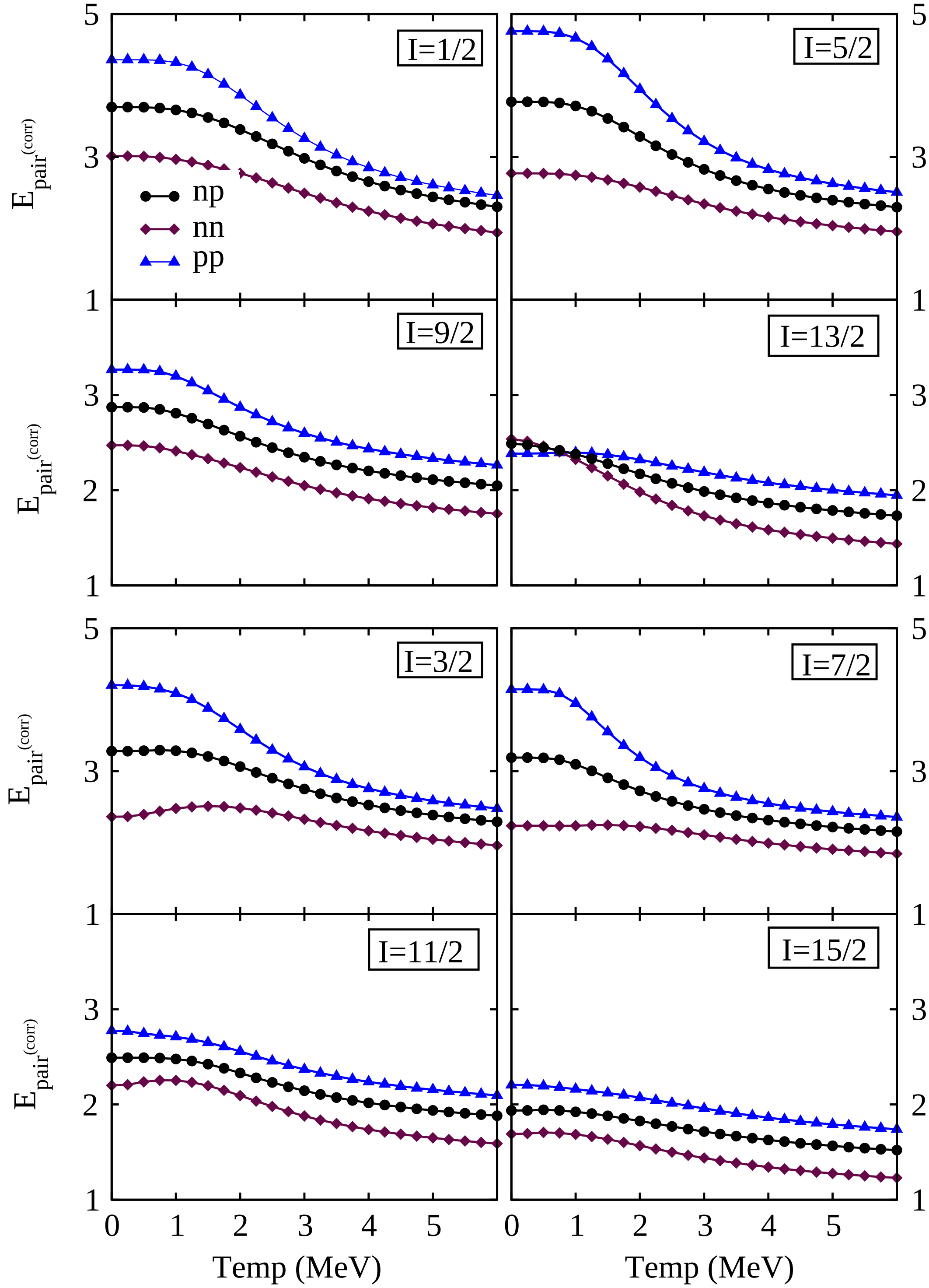

FIG. 3: (color online)Temperature dependence of pairing energy for ^{27}Si. The results are shown for I = 1/2, 5/2, 9/2 and 13/2 on the upper part and for I = 3/2, 7/2, 11/2 and 15/2 on bottom panels.

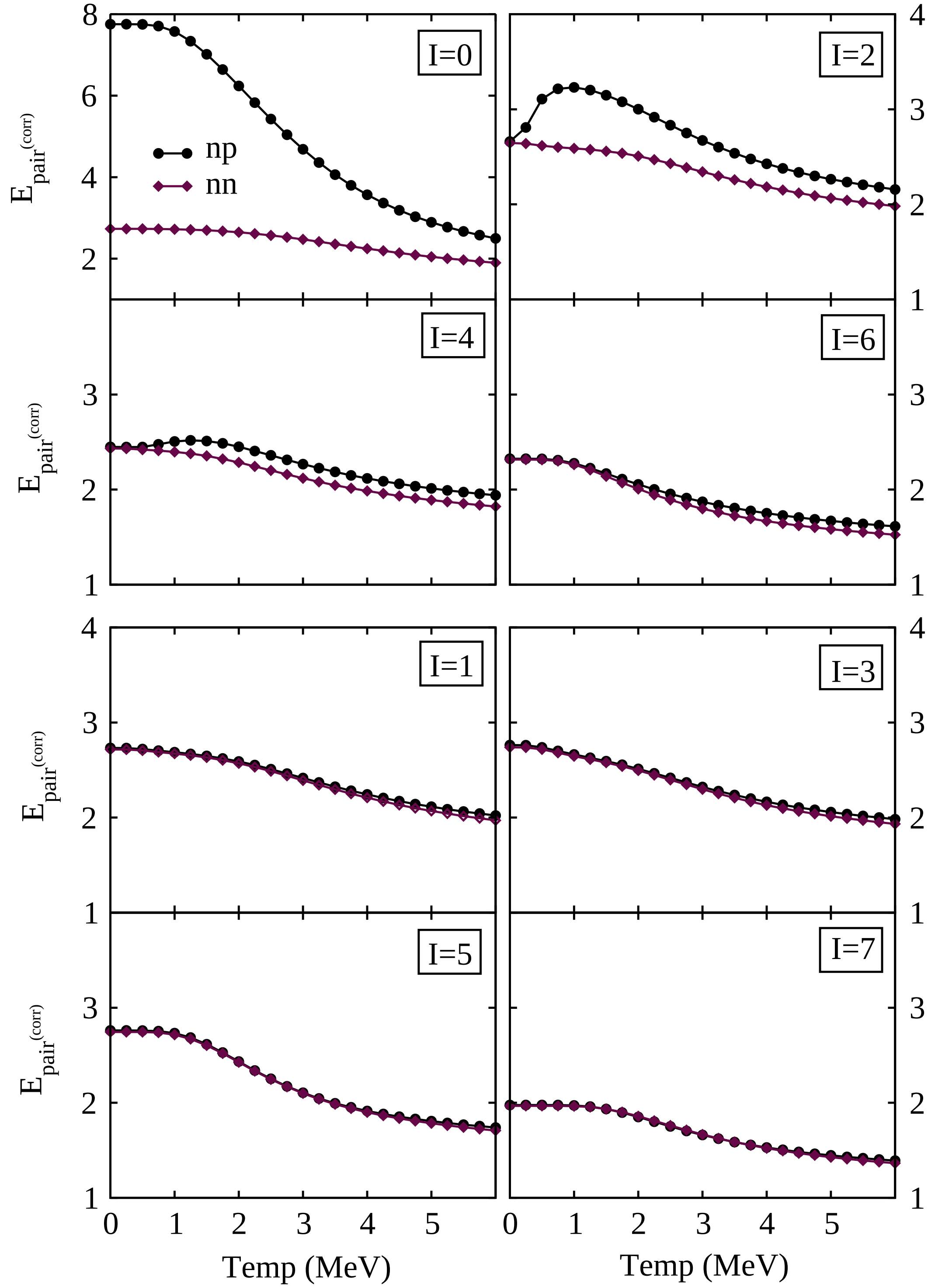

FIG. 4: (color online) Temperature dependence of pairing energy for ^{26}Al. The pair-gaps are plotted for I = 0, 2, 4,and 6 on upper two panels and for I = 1, 3, 5,and 7 on the lower two panels.

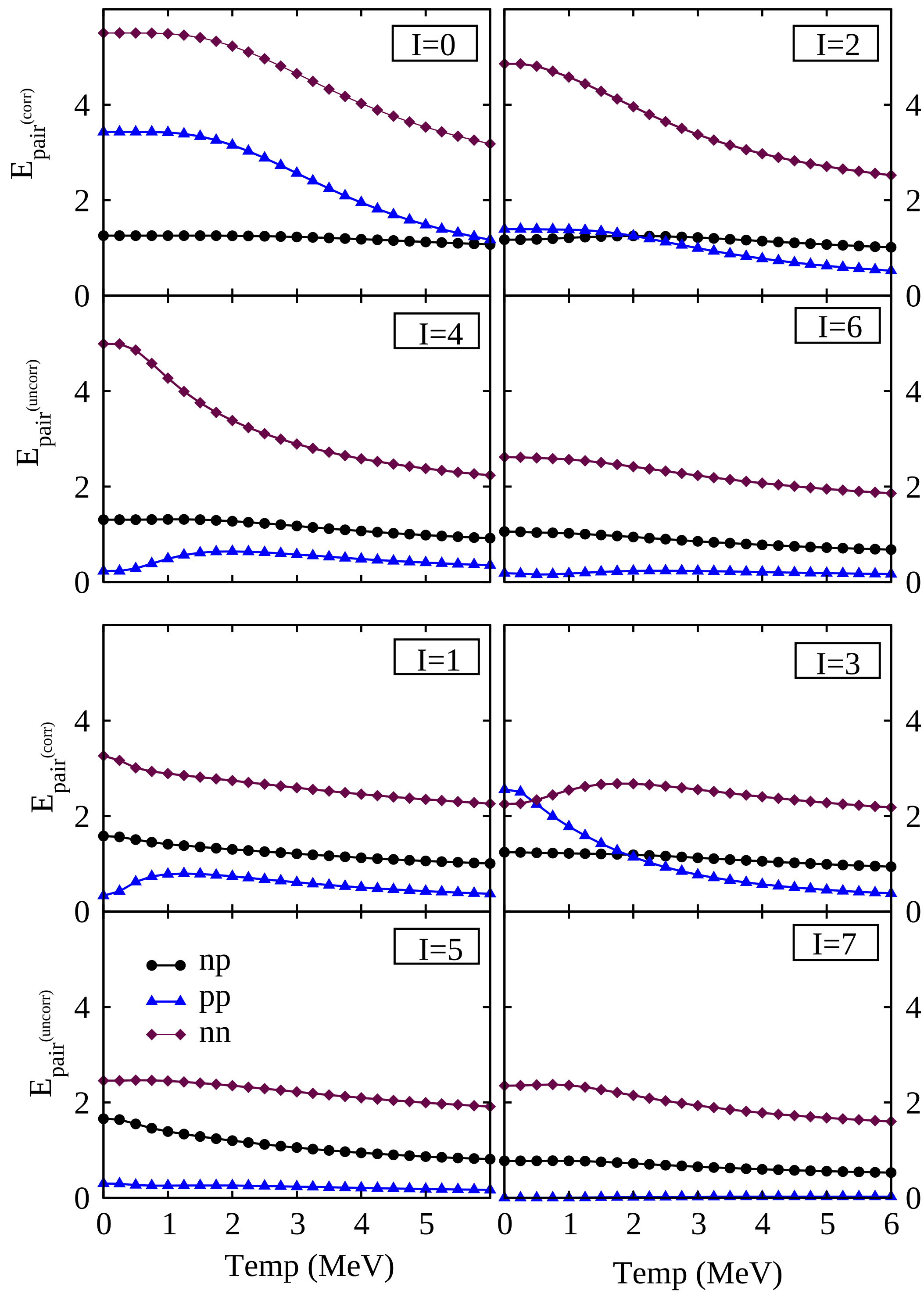

FIG. 5: (color online)Temperature dependence of pairing energy for ^{24}Ne.The pair-gaps are plotted for I = 0, 2, 4,and 6 on upper two panels and for I = 1, 3, 5,and 7 on the lower two panels.

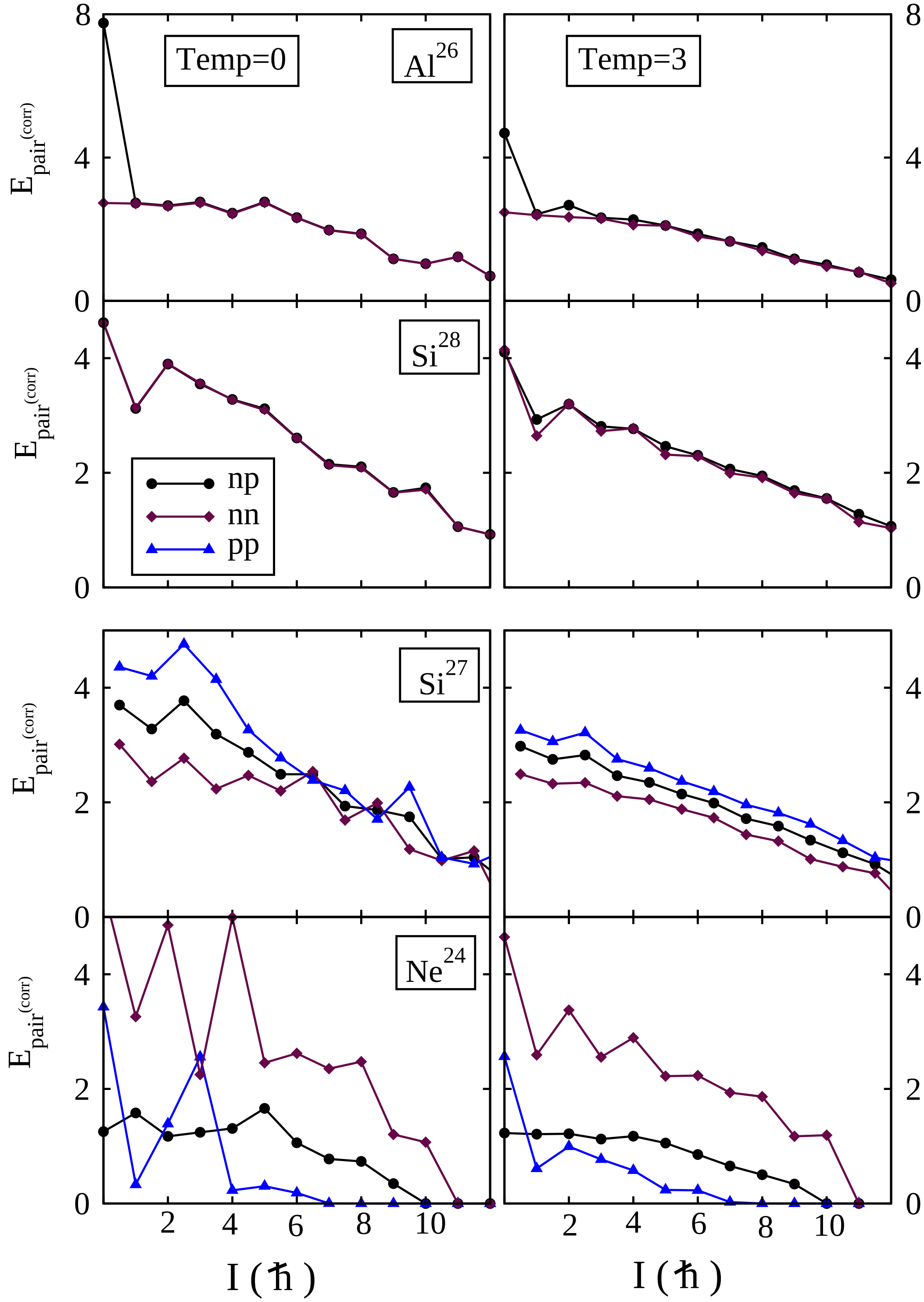

FIG. 6: (color online) Angular momentum dependence of the pairing energy for ^{26}Al,^{28}Si,^{27}Si,and ^{24}Ne for two different values of temperature.

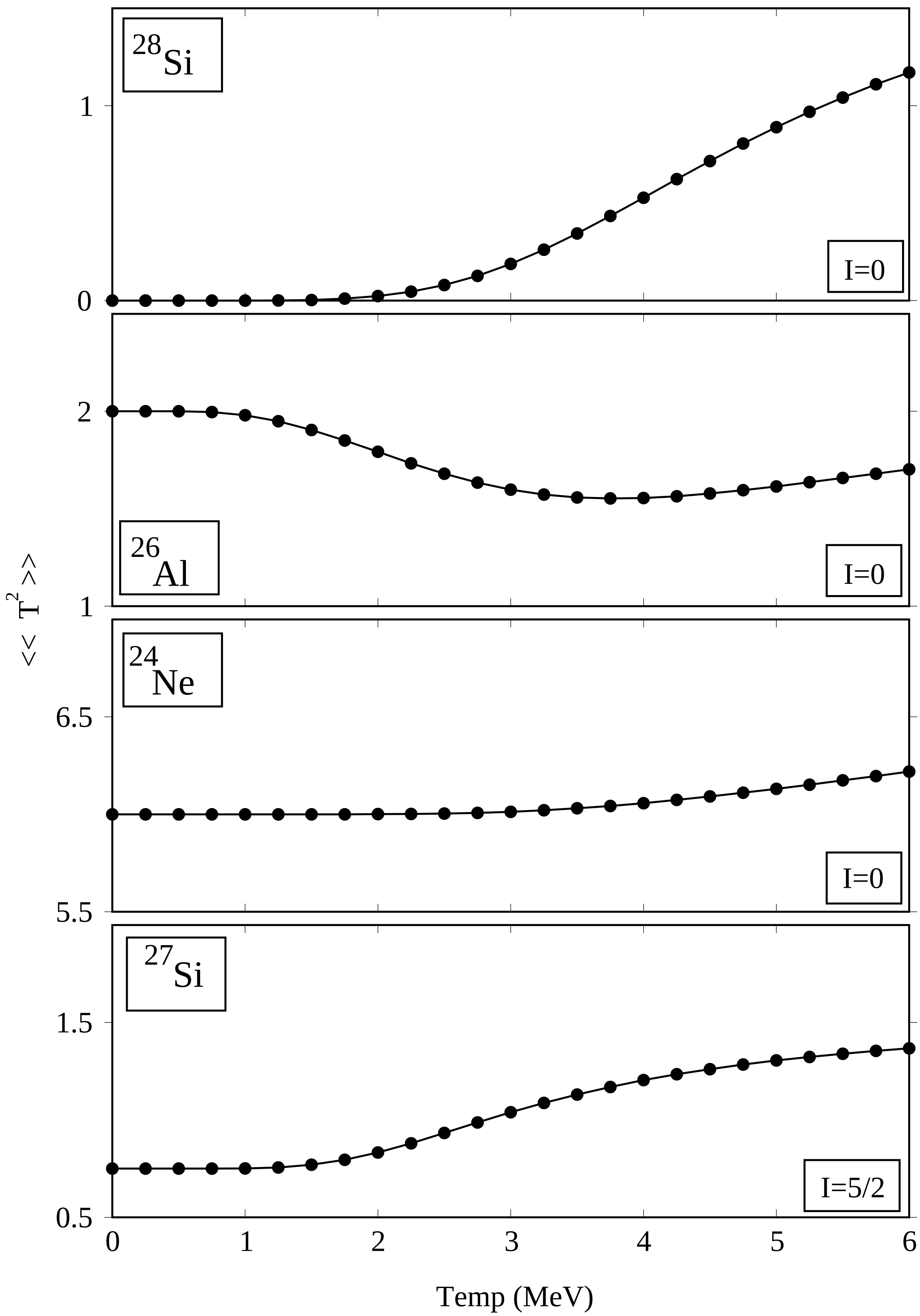

FIG. 7: (color online) Dependence of average isospin on temperature for the lowest angular momentum ensembles of ^{28}Si,^{26}Al,^{24}Ne and ^{27}Si.